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### Radial consolidation modelling incorporating downdrag effect for a multi-layer soil

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**RADIAL CONSOLIDATION MODELLING INCORPORATING THE EFFECT  
OF SMEAR ZONE FOR A MULTI-LAYER SOIL WITH DOWNDRAG OF  
CAUSED BY MANDREL ACTION**

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# **RADIAL CONSOLIDATION MODELLING INCORPORATING EFFECT OF THE SMEAR ZONE FOR A MULTI-LAYER SOIL WITH DOWNDRAG OF SOIL CAUSED BY MANDREL ACTION**

Cholachat Rujikiatkamjorn and Buddhima Indraratna

## **Abstract**

A system of prefabricated vertical drains with surcharge preloading is an effective method for promoting radial drainage and accelerated soil consolidation. A piecewise technique is employed to analyse the radial consolidation in the multi-layer soil system to include (a) the effect of soil downdrag and (b) smear zone having soil permeability varying linearly. The effect of soil dragged down from the upper soil layer into the lower layer has been analysed in terms of required time for consolidation. It can be seen that the consolidation of the multi layer soil depends on smear zone characteristics, permeability ratio between upper and lower soil layers, penetration depth, and drain spacing. Design procedures are described with a worked out example.

Key words: Consolidation, Design charts, Multi-layer soil, Smear zone, Vertical drains.

## Introduction

Due to rapid coastal development in Australia, most infrastructure is forced to be built on marine or estuarine soft clays that are highly compressible and of low shear strength, unless substantially consolidated prior to infrastructure construction. The coastal areas contain thick layers of compressible clay which is highly compressible and low permeable. Ground improvement technique is required before the commencement of the permanent structures. Preloading of soft clay with prefabricated vertical drains (PVDs) is one of the most common methods for improving the shear strength of soil and to reduce its post-construction settlement. Using PVDs, the drainage path length is reduced significantly from the soil layer thickness to half of the drain spacing. Moreover, the horizontal hydraulic conductivity is usually larger than the vertical hydraulic conductivity, hence, the consolidation process can occur much faster (Jamiolkowski et al. 1983). This system has been used successfully to improve foundation soils for airports, rail and road embankments (Indraratna and Redana 2000; Li and Rowe 2002).

PVDs are normally installed using steel mandrel pushing through soft ground creating smear zone surrounding PVDs. The soil in the smear zone is disturbed with an associated permeability reduction, which is detrimental to radial consolidation. The extent of smearing depends on the mandrel size, installation procedure and soil type (Eriksson et al., 1999; Lo, 1998; Bo et al. 2003). Highly sensitivity clays with prominent macro fabric generally exhibit the greatest smear effects. A number of studies have noted that the disturbance in the smear zone increases towards the drain (Chai and Miura, 1999; Hawlader et al., 2002; Sharma and Xiao, 2000; Indraratna and Redana, 2000; Madhav et al., 1993; Bergado et al., 1991). Laboratory studies on circular sand drains and rectangular PVD illustrate a linear decrease in horizontal permeability towards the drain. The permeability close to the drain can be decreased by one order of magnitude (Bo et al., 2003) and is usually assumed to be the same as the vertical permeability (Hansbo, 1981; Indraratna and Redana, 2000). The vertical permeability remains relatively unchanged. In layered soil, the disturbance of the lower layer is not only due to the displacement of soil surrounding PVDs but also the down-dragged disturbed soil in the upper layer forming an additional smear zone (Casagrande and Poulos, 1969; Hird and Moseley, 2000).

Barron (1948) introduced the radial consolidation theory including the effects of smear (for equal strain) and well resistance (for equal and free strain). These solutions involved Bessel functions that were time consuming to perform. Consolidation solutions for radial drainage were further developed including: rigorous solutions for vertical and horizontal drainage for equal strain with well resistance (Yoshikuni and Nakanodo, 1974, Onoue et al., 1988). Hansbo (1981) proposed a much simpler solution incorporating both smear effect and well resistance into Barron's formulation. Indraratna et al (2005a, b) incorporated the effect of vacuum pressure and non-linear soil compressibility and permeability for radial consolidation. In the above solutions, a reduced permeability is simply assumed as a constant. Subsequently, Walker and Indraratna (2007) and Basu and Prezzi (2007) employed a linear decay in horizontal permeability towards the drain representing a more realistic variation of soil permeability within the smear zone.

A number of solutions exist for layered soils of which Zhu and Yin (2005) presented design charts for vertical drainage with two layers. Xie et al. (1999) solved the same problem with partially drained boundaries, while Xie et al. (2002) incorporated the small strain theory and nonlinear soil properties where the decrease in permeability is assumed proportional to the decrease in compressibility (i.e. coefficient of consolidation is constant). Double layered ground with radial and vertical drainage was studied by Tang and Onitsuka (2001) and Wang and Jiao (2004). However, none of them considered the effect of soil dragged down from the upper layer to create additional smear in the underlying layer.

A piecewise-linear technique is a method that divides the soil properties into small segments based on material coordinates so that the soil properties can be considered as a constant for a particular segment. This method offers a more general solution compared to the exact solutions that can be applicable for any particular conditions (Walker 2006). Fox et al. (2003) has applied this method for radial consolidation accounting for vertical strain and flow, soil self-weight, and variable hydraulic conductivity and compressibility during the consolidation process.

In this study, a radial consolidation model using a piecewise technique will be proposed to study the smear effect in layered soil. The effects of permeability of the

penetrating upper soil layer on the underlying soil layer are discussed in terms of the degree of consolidation. The intrusion of the upper soil layer into the underlying soil layer creates an additional zone where the remoulded permeability of this zone can be increased or decreased depending on the initial permeability of the upper soil layer. In the intrusion zone located in the lower soil layer, the permeability variation can be divided into 3 zones including (a) smear zone due to the dragdown of the upper soil layer, (b) smear zone due to the remoulding of the underlying soil, and (c) the undisturbed lower soil layer. So far there is no technique to take this effect into consideration. The existing theory can only be used for the unit cell with a smear zone and the undisturbed zone. The piecewise-linear technique has been considered as an appropriate approach to determine the effect of soil dragdown on the overall consolidation. An array of design charts with a worked-out example is illustrating the role of down-dragged soil.

### **Theoretical Development**

The main assumptions for radial consolidation considering equal vertical strain condition are assumed to be applicable to a unit cell (Hansbo 1981), and they are:

- (1) Laminar flow through the saturated and homogeneous soil (Darcy's law) is valid. Flow is not allowed to occur at the outer boundary (Fig. 1), and for relatively long vertical drains (drain length is usually more than 10 times the drain spacing), only the lateral flow is permitted to occur (i.e. no vertical flow) (Rujikiatkamjorn and Indraratna, 2007).
- (2) Soil deformation is uniform at the upper boundary of the unit cell and the small strain theory is valid.
- (3) Based on the equal strain concept (Barron 1948, Indraratna et al. 2005a, Walker and Indraratna 2007) that all vertical strains at any given depth  $z$  are assumed to be equal, as the radial consolidation prevails.

To enable the use of index notation the inner radius  $r_w$  and outer radius  $r_e$  have been replaced with  $r_0$  and  $r_m$  respectively. Figure 1 shows a unit cell with an external radius  $r_e$ , drain radius  $r_w$ , and an initial drainage path length  $l$ . Any radial distribution

of lateral permeability ( $k_h$ ) can be approximated by dividing the soil into  $m$  segments as in Fig. 2.  $k_h$  is constant within each segment.

Considering flow in segment  $i^{\text{th}}$  (Fig. 1), the pore pressure gradient in the  $i^{\text{th}}$  segment is given by:

$$[1] \quad \frac{\partial u_i}{\partial r} = \frac{\gamma_w r_m^2}{2\bar{k}_h} \frac{\partial \varepsilon}{\partial t} \kappa_i \left[ \frac{1}{r} - \frac{r}{n^2 r_0^2} \right]$$

where,

$$[2] \quad \kappa_i = \frac{\bar{k}_h}{k_{hi}}$$

$$[3] \quad n = \frac{r_m}{r_0}$$

In the above,  $\varepsilon$  = average vertical strain at depth  $z$ ,  $c_h$  = horizontal coefficient of consolidation  $u$  = excess pore pressure

The permeability ratio,  $\kappa_i$ , is calculated with respect to a convenient reference value,  $\bar{k}_h$  (usually that of the undisturbed soil).

Using the boundary conditions  $u(r_0) = w$  and  $u_i(r_i) = u_{i+1}(r_i)$ , Equation (1) is integrated in the  $r$  direction for each segment to give the pore pressure in the  $i^{\text{th}}$  segment as:

$$[4] \quad u_i = -\frac{r_m^2}{2c_h} \frac{\partial \bar{u}}{\partial t} \left[ \kappa_i \left( \ln \left( \frac{r}{r_{i-1}} \right) - \frac{1}{2} \left( \frac{r^2}{n^2 r_0^2} - \frac{s_{i-1}^2}{n^2} \right) + \psi_i \right) + \frac{k_{hi}}{q_w} \pi z (2l - z) \left( 1 - \frac{1}{n^2} \right) \right]$$

where,

$$[5] \quad s_i = \frac{r_i}{r_0}$$

and, [6] 
$$\psi_i = \frac{1}{\kappa_i} \sum_{j=1}^{i-1} \kappa_j \left[ \ln \left( \frac{s_j}{s_{j-1}} \right) - \frac{1}{2} \left( \frac{s_j^2}{n^2} - \frac{s_{j-1}^2}{n^2} \right) \right]$$

The pore water pressure in the drain at depth  $z$  is designated as  $w(z)$ . For vertical flow in the drain, the change in flow from the entrance to the exit face of the slice with thickness  $dz$  (Fig. 1) is given by:

$$[7] \quad dQ_1 = -\frac{\pi r_w^2 k_w}{\gamma_w} \left( \frac{\partial^2 w(z)}{\partial z^2} \right) dz dt$$

where,  $k_w$  = drain permeability.

The radial flow into the slice is then determined from:

$$[8] \quad dQ_2 = \pi (r_e^2 - r_w^2) \frac{\partial \varepsilon}{\partial t} dz dt$$

Assuming no sudden drop in pore pressure at the drain-soil boundary (that is,  $u = w$  at  $r = r_w$ ), then for continuity,

$$[9] \quad dQ_1 = dQ_2$$

Substituting Equations (7) and (8) into Equation (9) and rearranging gives:

$$[10] \quad \frac{\partial^2 w(z)}{\partial z^2} = \frac{\gamma_w}{q_w} \pi (r_e^2 - r_w^2) \frac{\partial \varepsilon}{\partial t}$$

where,  $q_w$  is the discharge capacity of the drain given by:

$$[11] \quad q_w = k_w \pi r_w^2$$

Integrating Equation (10) in the  $z$  direction with the boundary conditions  $w(0) = 0$  and  $w(2l) = 0$ , reveals the pore water pressure in the drain to be:

$$[12] \quad w(z) = \frac{r_e^2 \gamma_w}{2q_w} \pi (2l - z) \left( 1 - \frac{1}{n^2} \right) \frac{\partial \varepsilon}{\partial t}$$

The average pore water pressure  $\bar{u}$ , and the pore pressure distribution with radius are related by the algebraic expression:

$$[13] \quad \pi (r_e^2 - r_w^2) \bar{u} = \int_{r_w}^{r_e} 2\pi \sum_{i=1}^m \left( \frac{r_i u_i}{m} \right) dr$$



Performing the integration in Equation (13), the resulting expression for  $\bar{u}$  can be rearranged in the following form:

$$[14] \quad \bar{u} = \frac{\gamma_w r_e^2}{2\bar{k}_h} \frac{\partial \varepsilon}{\partial t} (\mu + \mu_w)$$

the  $\mu$  parameter is revealed as:

$$[15] \quad \mu = \frac{n^2}{n^2 - 1} \sum_{i=1}^m \kappa_i \left[ \frac{s_i^2}{n^2} \ln \left( \frac{s_i}{s_{i-1}} \right) - \frac{1}{2} \left( \frac{s_i^2}{n^2} - \frac{s_{i-1}^2}{n^2} \right) - \frac{(s_i^2 - s_{i-1}^2)^2}{4n^4} + \psi_i \left( \frac{s_i^2}{n^2} - \frac{s_{i-1}^2}{n^2} \right) \right]$$

where,  $\mu_w$  is the contribution of the well resistance given by:

$$[16] \quad \mu_w = \frac{\bar{k}_h}{q_w} \pi z (2l - z) \left( 1 - \frac{1}{n^2} \right)$$

If well resistance is not included ( $q_w \rightarrow \infty$ ) then  $\mu_w$  is omitted. To give an approximate indication as to how the entire soil layer is affected by well resistance Equation (16) can be averaged over length  $l$  to give:

$$[17] \quad \mu_w = \frac{\bar{k}_h}{q_w} \frac{2\pi l^3}{3} \left( 1 - \frac{1}{n^2} \right)$$

The  $\mu$  parameter lies at the heart of the equal strain approach.  $\mu$  is a non-dimensional parameter depending only on the geometry and material property ratios of the soil/drain system.

The average degree of consolidation due to PVDs ( $U_h$ ) can be determined from:

$$[18] \quad U_h = 1 - \exp\left(\frac{-8T_h}{\mu}\right)$$

$$[19] \quad T_h = c_h t / d_e^2$$

where  $T_h$  = time factor,  $t$  = time,  $d_e$  = drain influence zone,

Hansbo's (1981) formulation for a smear zone with constant reduced permeability and an undisturbed zone is a special case of the multiple segment solution presented above.

For a single smear zone  $m = 2$ ,  $\kappa_1 = k_h/k'_h = \kappa$ ,  $\kappa_2 = 1$ ,  $s_0 = 1$ ,  $s_1 = r_s/r_w = s$  and  $s_2 = r_e/r_w = n$  where  $r_s$  is the smear zone radius.

The  $\mu$  parameter is then given by:

$$\begin{aligned} \mu_s = & \frac{n^2}{n^2 - 1} \left( \ln \frac{n}{s} + \kappa \ln s - \frac{3}{4} \right) + \frac{s^2}{n^2 - 1} \left( 1 - \frac{s^2}{4n^2} \right) + \\ [20] \quad & + \frac{\kappa}{n^2 - 1} \left( \frac{s^4 - 1}{4n^2} - s^2 + 1 \right) \end{aligned}$$

Ignoring higher order (insignificant) terms Equation (20) reduces to:

$$\mu_s = \ln \frac{n}{s} - \frac{3}{4} + \kappa \ln s \quad [21]$$

### Problem Definition for Multi-layer Soil

During the vertical drain installation using a steel mandrel, the soil surrounding the mandrel is dragged down by friction at the soil-mandrel interface. It is well known that mandrel action remoulds the soft clay causing a reduction in permeability. The high effective vertical stresses caused by the mandrel further decreases the lateral permeability. In addition, the mandrel action that forces soil to be dragged down from upper layers to the underlying regions distinctly divides the clay seam into multi-layer permeability zones (Fig. 3). The consolidation process of the underneath soil layer can be retarded, if the soil permeability in the upper layer is lower. As the dragdown of soil influences the consolidation of lower soil layer, a two-layer soil system can be used to determine the consolidation response of the lower layer (Fig. 4). The dragdown effect influences the variation pattern of soil permeability of underlying layer. In the analysis, the permeability of lower soil layer with the intrusion zone has been divided into 3 zones, the undisturbed zone, the smear zone due to the remoulding of lower soil layer and the smear zone due to the intrusion of the upper soil layer. Indraratna and Redana (2000) show that the lateral permeability can vary in the order of 10 within the multilayer soil in which the soil compressibility remains relatively constant, and the variation of the extent of the smear zone ( $d_s$ ) is insignificant. On the basis of the above, the assumptions can be made:

- (a) Horizontal permeabilities of soil in layers 1 (upper) and 2 (lower) are  $k_{h1}$  and  $k_{h2}$ , respectively. Based on linear decay in horizontal permeability towards the drain proposed by Indraratna and Walker (2007), the smear zone characteristics (i.e.  $s$  and  $\kappa$ ) in both layers remain the same.
- (b) Both soil layers have the same compressibility and overlapping of smear zone does not occur.

Figure 4c presents the variation of soil permeability after mandrel installation at Sections 1-1, 2-2 and 3-3. Sections 1-1 and 3-3 represent the normal situation when the soil is homogeneous. In Section 2-2 where the intrusion of the soil layer (radius  $d_i/2$ ) occurs (infiltration zone), the permeability variation can be expressed as:

$$[22] \quad k_h = k_{h,upper} \left[ \frac{A_1}{r_w} r + B_1 \right]; \quad d_w/2 < r < d_i/2$$

$$[23] \quad k_h = k_{h,lower} \left[ \frac{A_2}{r_w} r + B_2 \right]; \quad d_i/2 < r < d_s/2$$

$$[24] \quad A_1 = \frac{\kappa_{upper} - 1}{s - 1}$$

$$[25] \quad A_2 = \frac{\kappa_{lower} - 1}{s - 1}$$

$$[26] \quad B_1 = \frac{s - \kappa_{upper}}{s - 1}$$

$$[27] \quad B_2 = \frac{s - \kappa_{lower}}{s - 1}$$

As observed from Casagrande and Poulos (1969) and Hird and Moseley (2000), the diameter of intrusion as a function of depth ( $z$ ) in layer 2 can be expressed as:

$$[28] \quad \alpha = \frac{(d_s + d_w)}{2} [1 - z/H_2] \quad 0 < z < H_2$$

The normalised penetration depth ( $D$ ) can be defined as a function of the penetration length due to upper soil layer ( $d_2$ ) and the lower clay thickness ( $H_2$ ) :

$$[29] \quad D = d_2 / H_2$$

The normalised penetration depth represents how deep the upper soil layer can be dragged down into the immediately underlying soil layer. When  $D$  is more than 1 (the lower soil layer is quite thin compared to penetration length due to upper soil layer), the entire underlying soil is affected by the down-dragging. When  $D$  is less than unity, the down-dragged soil from upper layer partially penetrates the underlying soil layer. The value of  $D$  has to be obtained by thorough field investigations as it depends on the interaction between the soil and the mandrel. In order to analyse this problem, a piecewise-linear technique (Equation 15) will be used in conjunction with Equations (22)-(29) to obtain the parameter  $\mu$ . For modern drains which have a very high discharge capacity ( $>150\text{m}^3/\text{year}$ ), the well resistance is negligible.

### **Influence of downdrag of upper soil layer to the underlying soil layer**

To investigate the downdrag influence of upper soil layer, the soil layer 2 in Fig. 4b is divided horizontally into  $n$  small layers having thickness of  $\frac{H_2}{n}$ . In each small horizontal layer  $i^{\text{th}}$ , The  $\mu_i$  parameter can be determined using the proposed piecewise linear technique (Equations 15) and the variation of the permeability in the intrusion zone can be calculated based on Equations (22)-(29). The average degree of consolidation for soil layer 2 can be determined from:

$$[30] \quad U_h = \frac{1}{n} \sum_{i=1}^n \left[ 1 - \exp\left(-\frac{8T_h}{\mu_i}\right) \right]$$

It can be seen that  $\mu_i$  depends on smear zone characteristics ( $\kappa$  and  $s$ ), permeability ratio between upper and lower soil layers ( $k_{h,upper} / k_{h,lower}$ ), penetration depth ( $D$ ), and drain spacing ( $n$ ).

The overall time factor at 90% degree of consolidation ( $T_{h90}$ ) of soil layer 2 can be determined from:

$$[31] \quad 0.9 = \frac{1}{n} \sum_{i=1}^n \left[ 1 - \exp\left(-\frac{8T_{h90}}{\mu_i}\right) \right]$$

Figure 5 presents the variation of  $T_{h90}$  with normalised penetration depth ( $D$ ) where  $n$  and  $s$  are assumed to be 30 and 5, respectively. When the horizontal permeability in the

upper soil layer ( $k_{h,upper}$ ) is less than that in the underlying soil layer ( $k_{h,lower}$ ) (i.e.  $k_{h,upper}/k_{h,lower} < 1.0$ ),  $T_{h90}$  significantly increases with an increase in penetration depth, retarding the consolidation in the second soil layer. However, when  $k_{h,upper}/k_{h,lower} > 1.0$ ,  $T_{h90}$  slightly decreases and becomes constant after  $D$  is less than 2. This implies that the horizontal permeability of the overlying soil layer can accelerate or retard the consolidation of the underlying soil layer.

### Design Procedures for Multi-layer Soil

Several vertical drains design procedures applicable for a single layer soil use horizontal time factor ( $T_h$ ) vs. degree of consolidation curves ( $U_h$ ) to determine the drain spacing ( $S$ ) (Hansbo 1981; Zhu and Yin 2005; Rujikiatkamjorn and Indraratna 2007). The determination of the drain spacing can become a cumbersome task when the downdrag effect of upper soil layer combines with an iteration process. In this Section, various time factors at 90% degree of consolidation ( $T_{h90}$ ) are generated based on a linear smear zone concept, where  $2 \leq \kappa \leq 5$ ,  $0.1 \leq k_{h,upper}/k_{h,lower} \leq 0.5$ ,  $0.1 \leq D \leq 10$ ,  $3 \leq n \leq 60$  and  $2 \leq s \leq 17$  (Figs. 6-11). The required drain spacing ( $S$ ) for a given soil layer considering the downdrag effect of upper soil layer to achieve 90% degree of consolidation can be determined by:

- (a) Assume a drain influence zone of  $d_e$ ;
- (b) Determine  $d_w$  from  $d_w = 2(b+d)/\pi$  (Hansbo 1981);
- (c) Determine  $n_1$  from  $n = d_e/d_w$ ;
- (d) Based on  $n_1$ ,  $D$ ,  $\kappa$ ,  $k_{h,upper}/k_{h,lower}$ , and  $s$ , determine  $T_{h90}$  from Figs. 6-17.

Calculate  $d_e$  from  $d_{e2} = \sqrt{\frac{c_h t}{T_{h90}}}$  where  $c_h$  = radial coefficient of consolidation,  $t$  =

required time. Determine  $n_2$  from  $n = d_{e2}/d_w$ . If the above parameters are the same as those given in the Figures, extrapolation and interpolation can be used to find  $n_2$ ;

- (e) If  $n_1 \neq n_2$ , select  $n_3$  from  $n_3 = (n_2 + n_1)/2$ . Repeat procedures (d) and (e) until  $n_1 = n_2$

(f) Determine drain influence zone ( $d_e$ ) from  $d_e=1.13S$  for PVDs with square pattern installation and  $d_e=1.05S$  for PVDs with triangular pattern installation;

(g) The overall degree of consolidation of  $n$ -layer soil ( $U$ ) can be determined by

$$[32] \quad U_h = \frac{\sum_i^n U_{hi} \Delta h_i}{\sum_i^n \Delta h_i}$$

where,  $U_{hi}$  is the degree of consolidation of soil layer  $i^{\text{th}}$  and  $\Delta h_i$  is the thickness of soil layer  $i^{\text{th}}$ .

For the upper soil layer, the degree of consolidation based on linear smear zone can be determined from Eq. (18) where  $\mu$  can be expressed by (Walker and Indraratna 2007):

$$[33] \quad \mu_L = \ln\left(\frac{n}{s}\right) - \frac{3}{4} + \frac{\kappa (s_L - 1)}{s - \kappa} \ln\left(\frac{s}{\kappa}\right)$$

For the case when  $s_L = \kappa_L$  the  $\mu$  parameter is:

$$[34] \quad \mu = \ln\left(\frac{n}{s}\right) - 1.75 + s$$

### Worked-out Example

The above methodology is illustrated by the following example. The foundation soils are divided into 2 layers, whereby  $t = 1.2$  years and well resistance is ignored. The required soil parameters for normally consolidated clay for each soil layer are assumed to be:

#### Layer 1 (Upper layer):

$$c_{h1} = 0.5 \text{ m}^2/\text{year}, \kappa = 5, s_1 = 5, k_{h, \text{upper}} = 1 \times 10^{-10} \text{ m/s}, H_1 = 7\text{m}$$

#### Layer 2 (Lower layer):

$$c_{h2} = 1.0 \text{ m}^2/\text{year}, \kappa = 5, s_2 = 5, k_{h, \text{lower}} = 2 \times 10^{-10} \text{ m/s}, H_2 = 3\text{m}, D = 0.5$$

PVDs with  $d_w = 0.06$  m are expected to be installed in square pattern

Determine the drain spacing ( $S$ ) when degree of consolidation of layer 2 is 90% and overall degree of consolidation for both upper and lower soil layer.

*Solution:*

Considering second soil layer

- a) Assume  $d_e = 0.9\text{m}$ ;
- b)  $n_1 = 0.9/0.06 = 15$ ,  $k_{h,upper} / k_{h,lower} = 0.5$ ;
- c) Based on  $n_1$ ,  $D$ ,  $\kappa$ ,  $k_{h,upper} / k_{h,lower}$ , and  $s$ , determine  $T_{h90} = 1.46$  from Fig. 11.
- d)  $n_2 = \sqrt{\frac{1 \times 1.2}{1.46}} / 0.06 = 15.1$ ,  $n_1 \approx n_2$ , therefore  $n = 15.0$
- e) Based on  $n_3$ ,  $D$ ,  $\kappa$ ,  $k_{h,upper} / k_{h,lower}$ , and  $s$ , determine  $T_{h90} = 1.48$  from Fig. 14.
- f)  $n_4 = \sqrt{\frac{1 \times 1.2}{1.48}} / 0.06 = 15.0$ , therefore  $n = 15$  and  $d_e = 0.9\text{m}$
- g) For the upper soil layer, the degree of consolidation determined from Eqs. (20) and (34) is 75%.
- h) The average degree of consolidation for both soil layers based on Equation (32) is 80%. For comparison, the overall degree of consolidation is 92% if the downdrag effect of soil is ignored in the lower layer.

## Conclusions

Prefabricated vertical drains increase the rate of soil consolidation by providing a short horizontal drainage path, and are now used worldwide in many soft soil improvement projects. A radial consolidation model using a piecewise technique is proposed to study the smear effect in a layered soil with the downdrag of soil caused by mandrel action. The consolidation of the multi-layer soil depends on smear zone characteristics, permeability ratio between upper and lower soil layers, penetration depth, and drain spacing. When the horizontal permeability in the upper soil layer is less than that of the underlying soil layer, consolidation time of the underlying soil layer can be retarded depending on the upper soil penetration depth. On the other hand, the consolidation process can be accelerated further when the downdrag upper soil layer has a higher permeability. The design charts have been provided to include this downdrag effect of the upper soil layer into immediate lower soil layer. The proposed design method is

most beneficial to the practitioner as a preliminary tool for the design of embankments stabilized by prefabricated vertical drains in a multi-layered soil affected by the downdrag.



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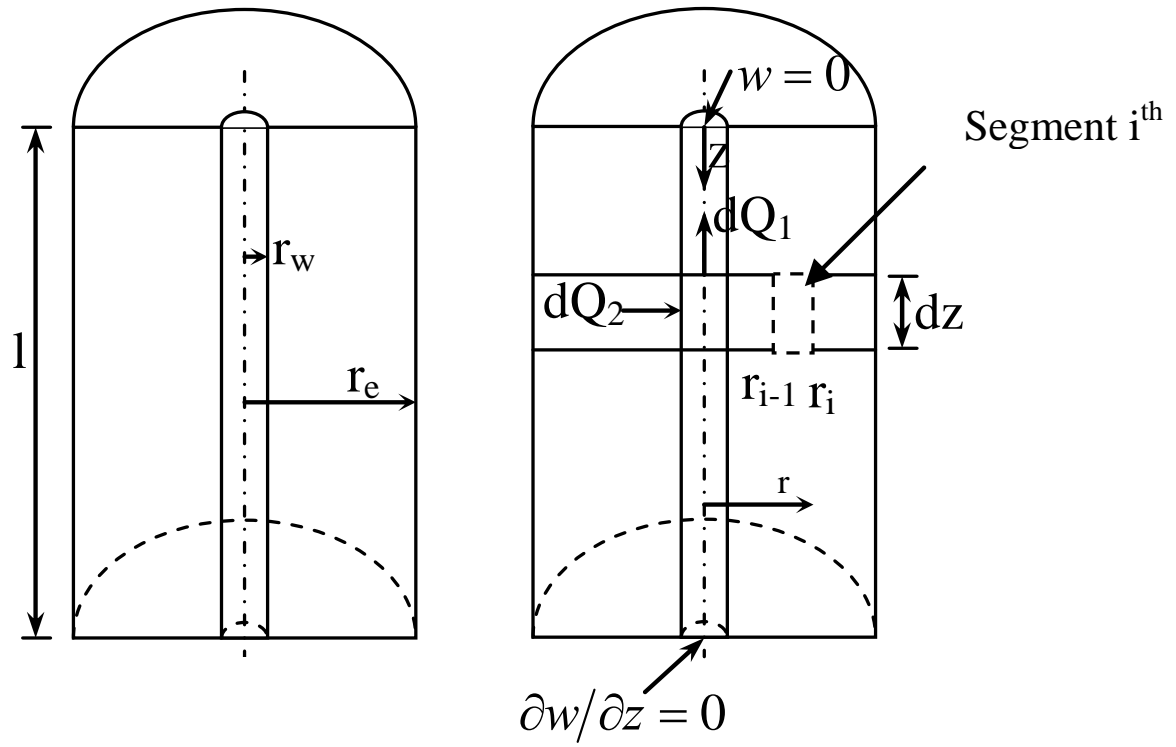
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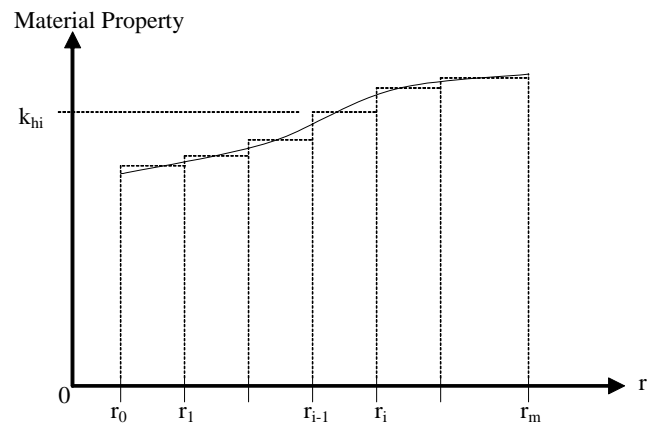
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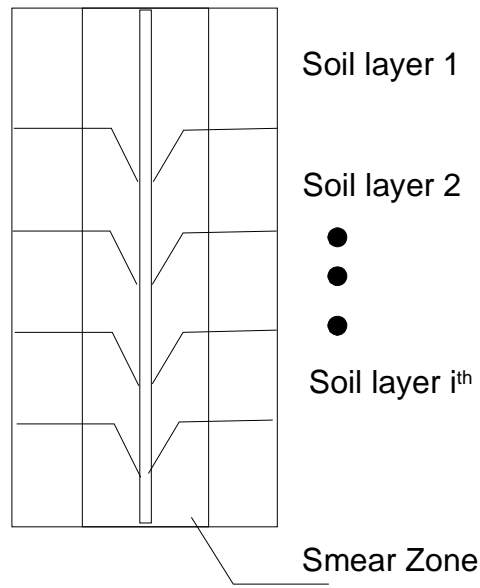
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**Figure 1 Axisymmetric unit cell**



**Figure 2 Discretised radial permeability**



**Figure 3 Downdrag effects due to mandrel installation in layered soil**





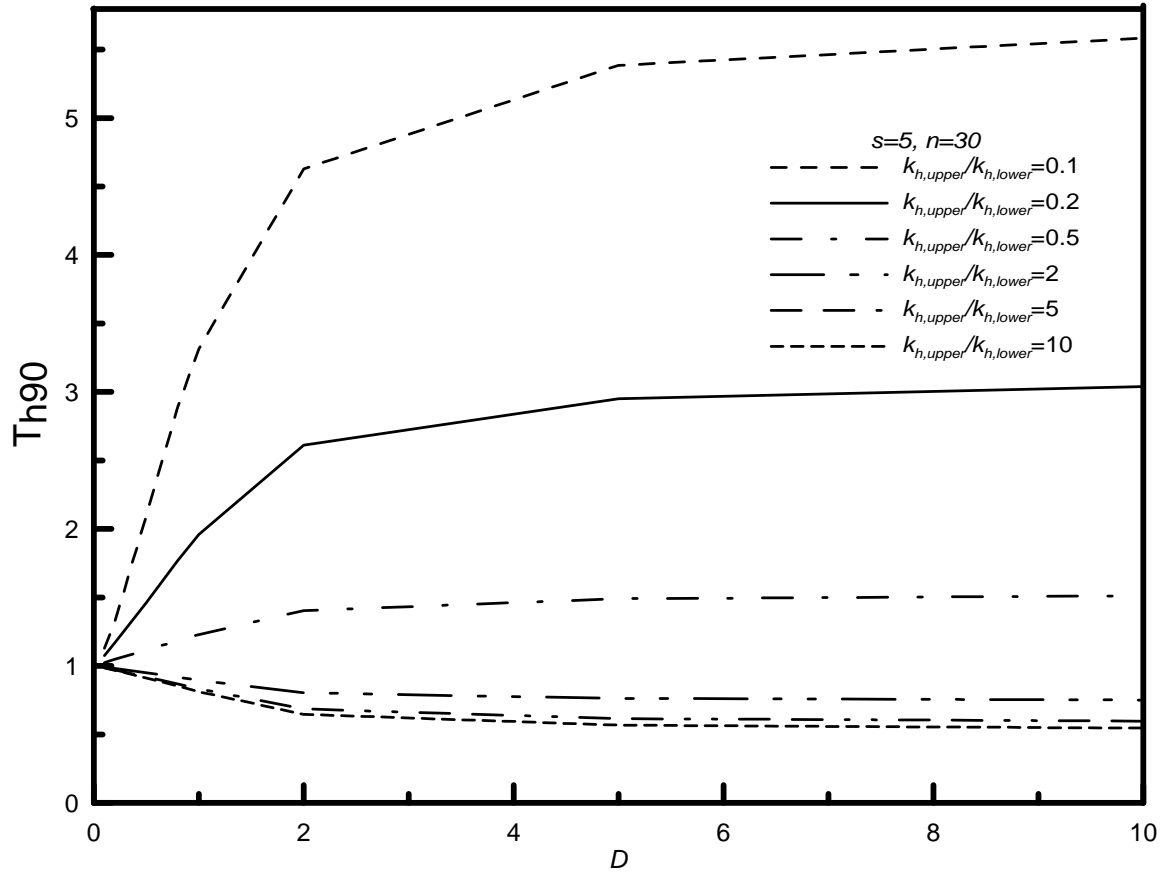


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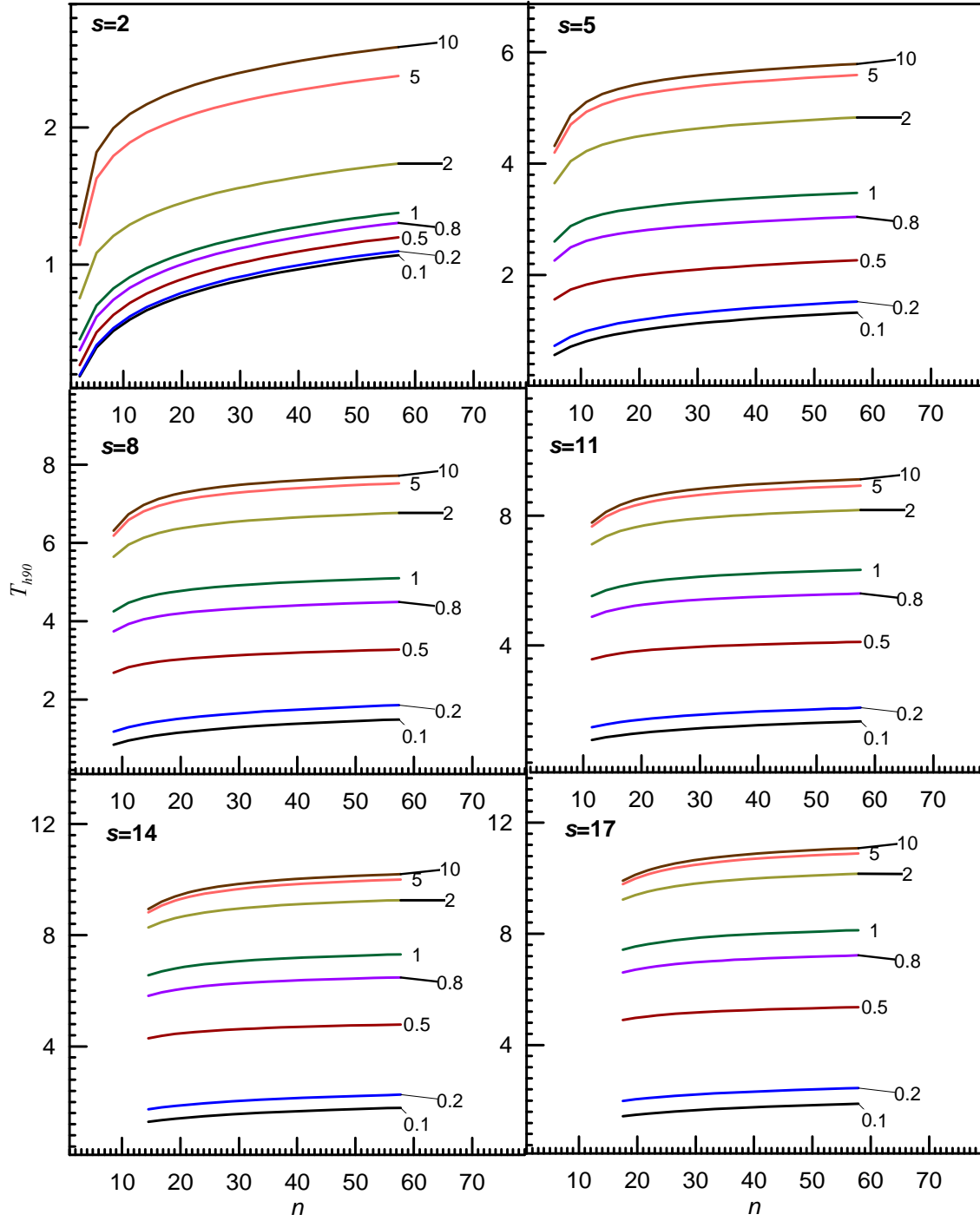


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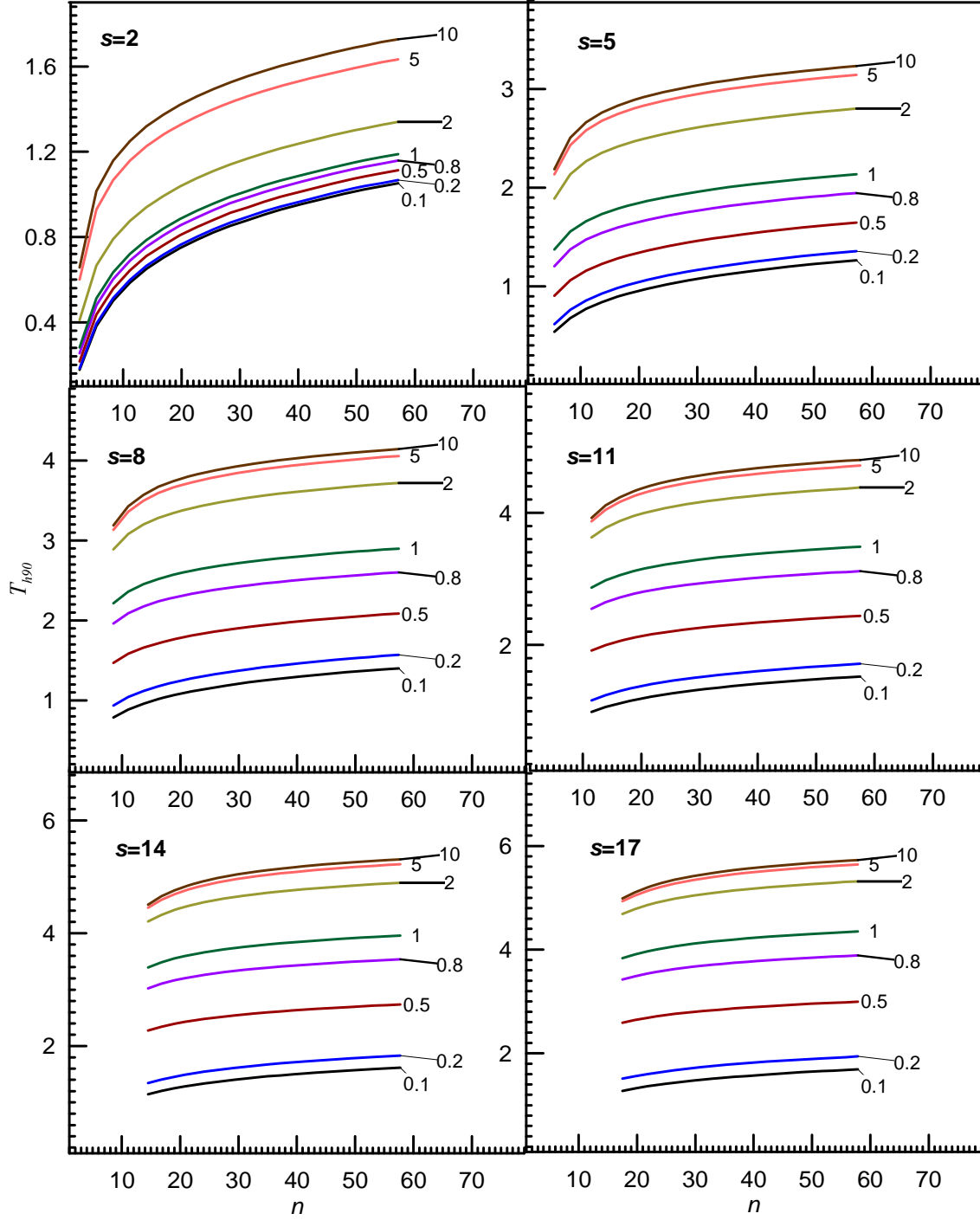


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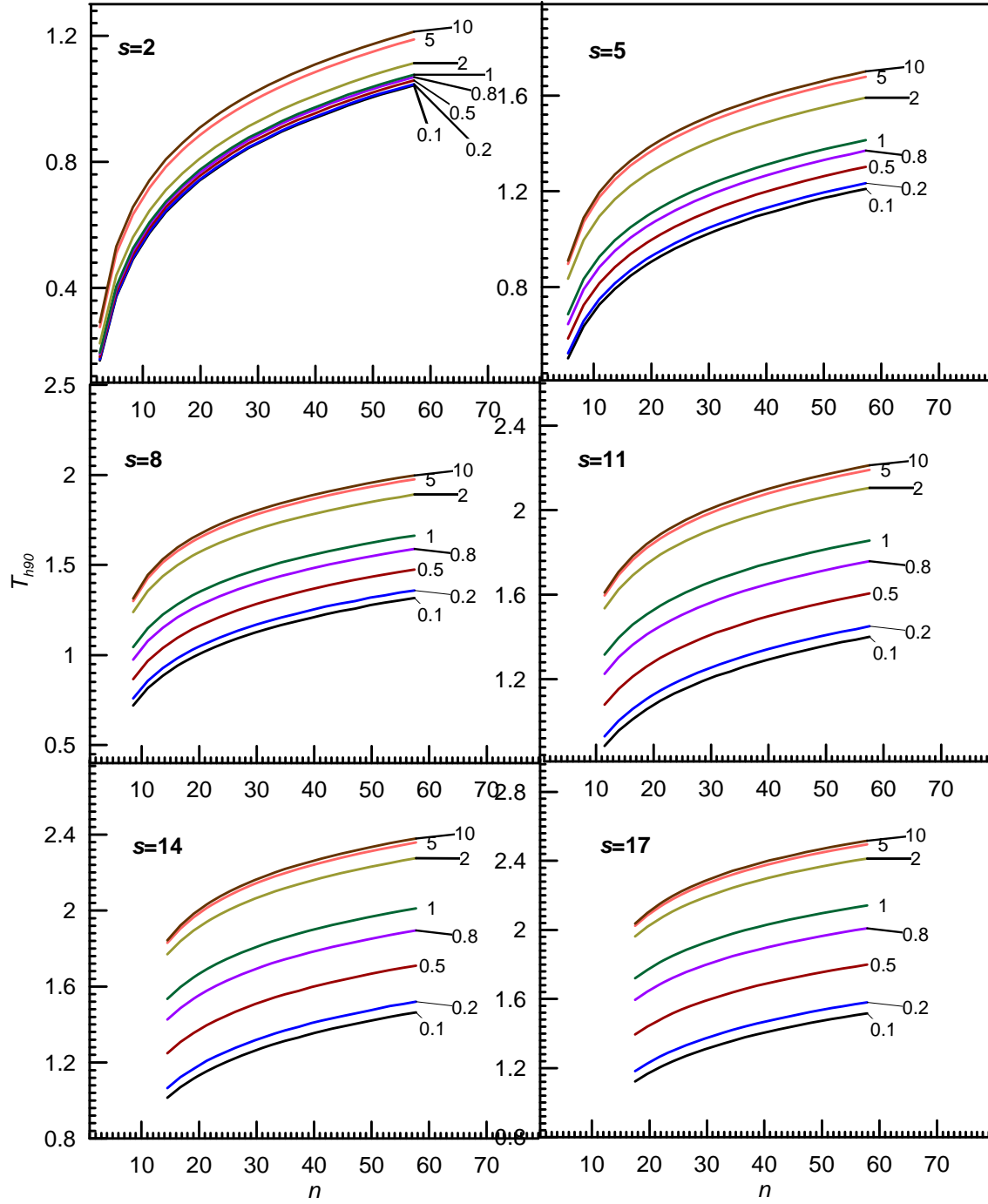


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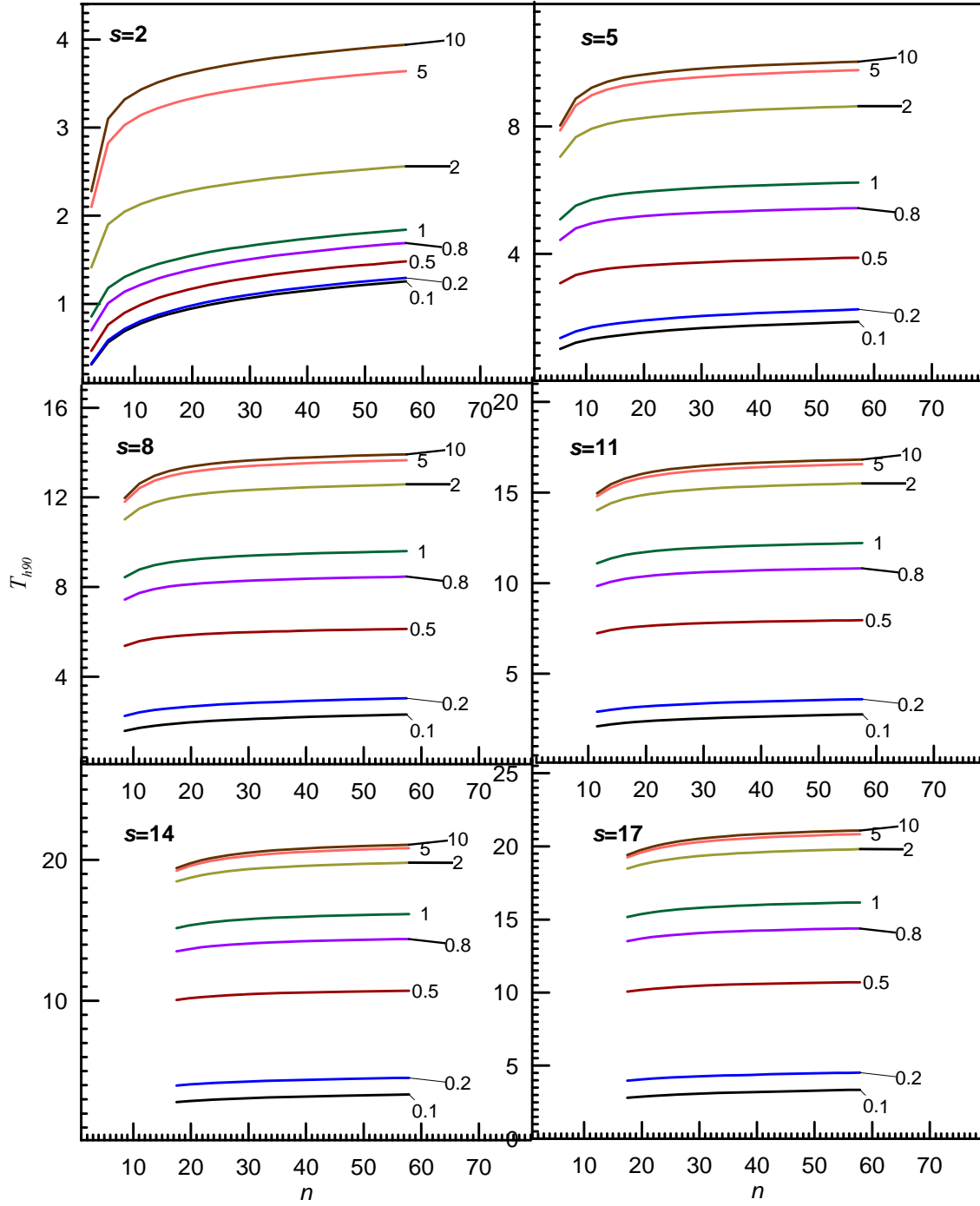


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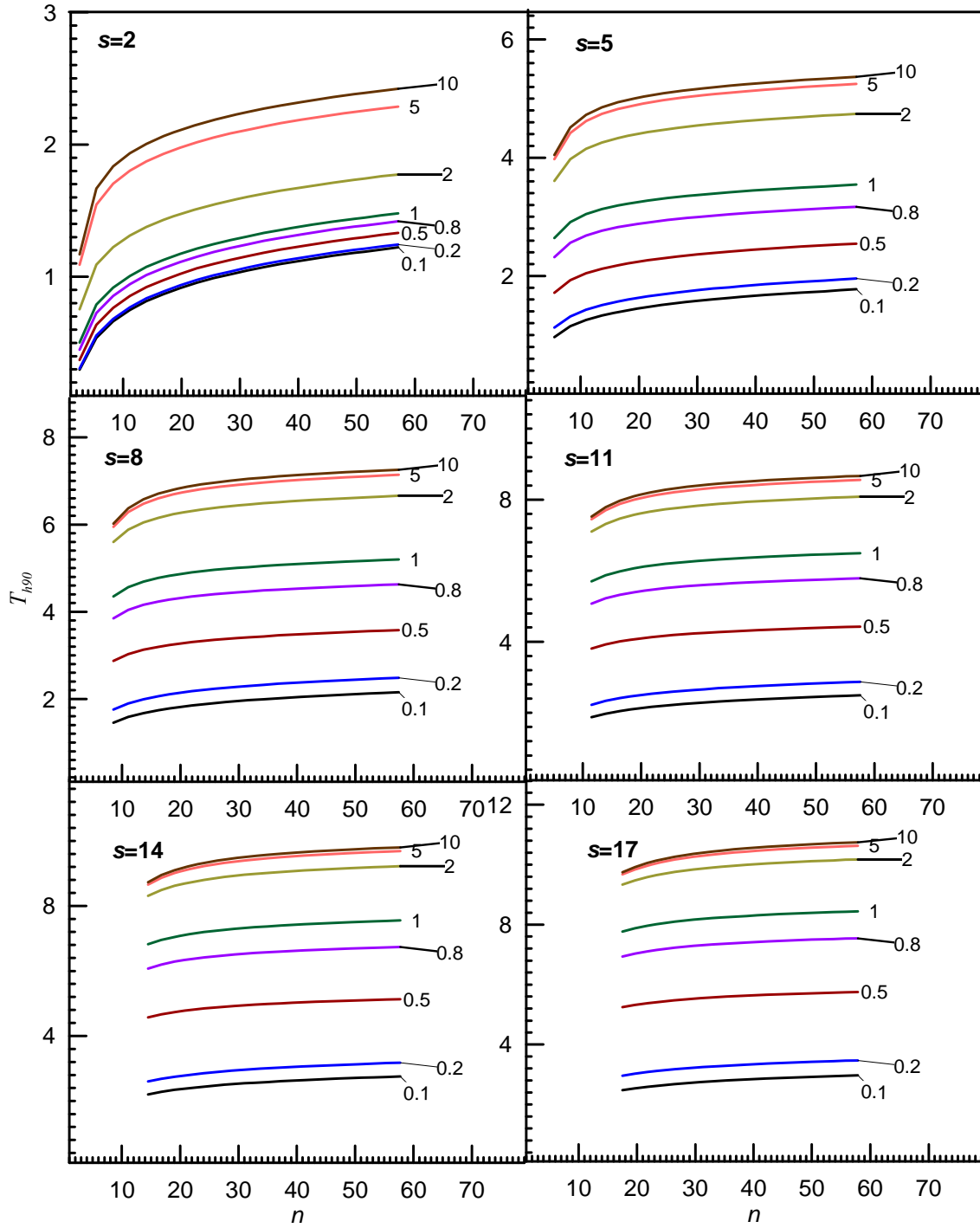


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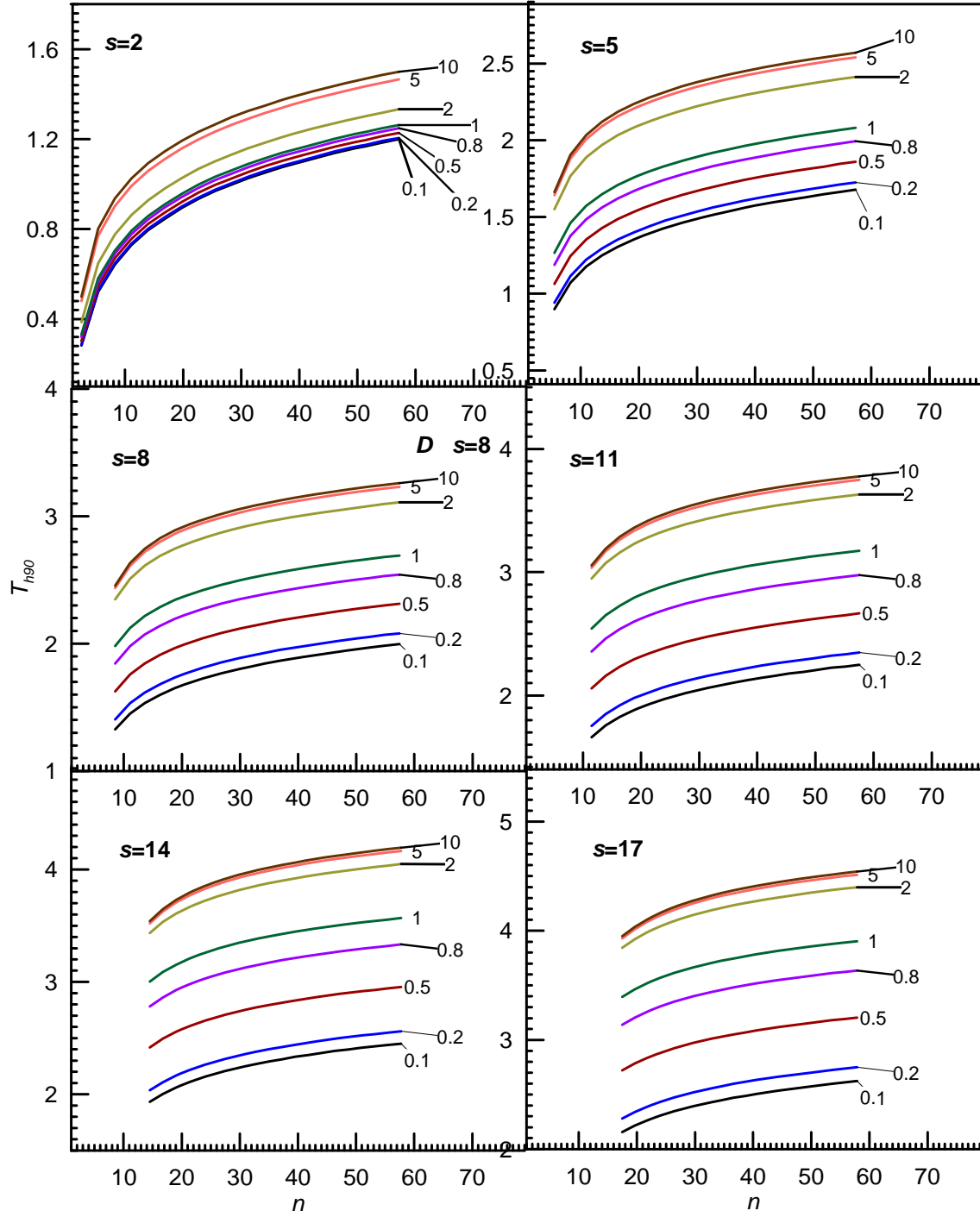


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